Nonperturbative calculations in SU(3) gauge theory

Vladimir Dzhunushaliev*, Douglas Singleton† and Tatyana Nikulicheva*

†Physics Dept., CSU Fresno, M/S MH37 Fresno, CA 93740-8031, USA.

Modern perturbative quantum field theories (QFT) are based on the concept of a particle or field quanta. This perturbative paradigm gives the most accurate description for some aspects of the physical world (e.g. the g-factor of the electron). Mathematically the elementary particles that arise in perturbative quantum field theory are quantized harmonic excitations of fundamental fields. The quanta are defined through the creation and annihilation operators, $a^\dagger$ and $a$, of the “second-quantized” theory.

The physical reason of the appearance of quanta is that in linear theories, such as electrodynamics, a general solution can be obtained by making the Fourier expansion:

$$A^{\text{linear}}_\mu = \int d^3 k \sum_{\lambda=0}^3 [a_k(\lambda) \varepsilon_\mu(k,\lambda) e^{-ik \cdot x} + a^\dagger_k(\lambda) \varepsilon^*_\mu(k,\lambda) e^{ik \cdot x}], \quad (1)$$

where $\varepsilon_\mu$ is the polarization vector. The key point is that the general solution can be obtained by a linear superposition of the plane wave solutions of the theory.

For a theory based on a non-Abelian group, like quantum chromodynamics (QCD), this can no longer be done in the strong coupling limit, due to the nonlinear nature of Yang-Mills equations. There are certain nonperturbative field configurations (for example, the hypothesized color electric flux tube that is thought to be important in the dual superconducting picture of confinement) in which the field distribution cannot be explained as a cloud of quanta. One way of looking at this situation is that the fields of these nonperturbative configurations are split into ordered fields (the fields inside flux tube stretched between quark and antiquark) and disordered fields (the fields outside the flux tube). These fields cannot be interpreted as a (perturbative) cloud of quanta. In such situations the fields play a primary role over the particles.

The need for nonperturbative techniques in strongly interacting, nonlinear quantum field theories is an old problem, and much effort has gone into trying to find an appropriate frame in which to carry on calculations for these theories. Despite this the problem is not yet fully resolved.

In [1] a possible approach was suggested based on a version of a quantization method originally due to Heisenberg. Starting with the classical SU(3) Yang-Mills equations

$$\partial_\nu F^{B\mu\nu} = 0 \quad (2)$$
\[ F^B_{\mu \nu} = \partial_\mu A^B_\nu - \partial_\nu A^B_\mu + gf^{BCD} A^C_\mu A^D_\nu \] is the SU(3) field strength) one replaces the classical fields by field operators \( \hat{A}^B_\mu \rightarrow \hat{A}^B_\mu \). This yields the following differential equations for the operators
\[ \partial_\nu \hat{F}^B_{\mu \nu} = 0. \] (3)

These nonlinear equations for the field operators of the nonlinear quantum fields can be used to determine expectation values for the field operators \( \langle \hat{A}^B_\mu (x) \rangle \), are obtained by averaging Eq. (3) over some quantum state \( |Q \rangle \)
\[ \langle Q \left| \partial_\nu \hat{F}^B_{\mu \nu} \right| Q \rangle = 0. \] (4)

One problem in using these equations to obtain expectation values like \( \langle \hat{A}^B_\mu \rangle \), is that these equations involve not only powers or derivatives of \( \langle \hat{A}^B_\mu \rangle \) (i.e. terms like \( \partial^\alpha \langle \hat{A}^B_\mu \rangle \) or \( \partial^\alpha \partial^\beta \langle \hat{A}^B_\mu \rangle \)) but also contain terms like \( \hat{g}^{BC}_{\mu \nu} = \langle \hat{A}^B_\mu \hat{A}^C_\nu \rangle \). Starting with Eq. (4) one can generate an operator differential equation for this product \( \hat{A}^B_\mu \hat{A}^C_\nu \) allowing the determination of the Green’s function \( \hat{g}^{BC}_{\mu \nu} \)
\[ \langle Q \left| \hat{A}^B (x) \partial_\nu \hat{F}^B_{\mu \nu} (y) \right| Q \rangle = 0. \] (5)

However this equation will in turn contain other, higher order Green’s functions. Repeating these steps leads to an infinite set of equations connecting Green’s functions of ever increasing order. This construction, leading to an infinite set of coupled, differential equations, does not have an exact, analytical solution and so must be handled using some approximation.

In order to do some calculations we give an approximate method which leads to the 2 and 4-points Green’s functions only [1]. We will consider two cases: in the first one the fields are in completely disordered phase. Starting with the pure SU(3) Lagrangian
\[ \hat{L}_{SU(3)} = \frac{1}{4} \hat{F}^B_{\mu \nu} \hat{F}^B_{\mu \nu} \] (6)

one can arrive at following effective, pure scalar Lagrangian (see [1] for details)
\[ \frac{g^2}{4} \mathcal{L}_{eff} = - \frac{1}{2} \left( \partial_\mu \phi^A \right)^2 + \frac{\lambda_1}{4} \left[ \phi^a \phi^a - \phi^a_0 \phi^a_0 \right]^2 + \frac{\lambda_2}{4} \left[ \phi^m \phi^m - \phi^m_0 \phi^m_0 \right]^2 + \left( \phi^a \phi^a \right) \left( \phi^m \phi^m \right) \] (7)

where the vector gauge fields have been replaced by effective scalar field, \( \phi^A \), through the ansatz
\[ \hat{g}^{BC}_{\alpha \beta}(x,y) \approx - \eta_{\alpha \beta} f^{BDE} f^{CDF} \phi^E (x) \phi^F (y) \] (8)

The \( a \) index in eq. (7) refers to the SU(2) subgroup of SU(3), and the \( m \) index refers to the coset SU(3)/SU(2), and two separate couplings \( (\lambda_1 \text{ and } \lambda_2) \) have been introduced.
between the SU(2) subgroup and the coset. A numerical investigation of eq. (7) yields a regular solution with finite energy. The profile of the energy density for this solution is given in the Fig. 1A. We interpret this solution as a glueball.

A second example involves both an ordered (described by a vector gauge field) and disordered (described by an effective scalar field) phase. Again after some assumptions and simplifications [1] we have find regular solutions with finite energy density. The effective Lagrangian in this case has the form

\[
\mathcal{L}_{\text{eff}} = -\frac{1}{4} h_{\mu \nu}^a h^{a \mu \nu} + \frac{1}{2} (D_\mu \phi^a)^2 - \frac{\lambda}{4} (\phi^a(x) \phi^a(x))^2 + \frac{g^2}{2} a^{b \mu} \phi^b a^{c \mu} \phi^c
\]

where \( h_{\mu \nu}^b = \partial_\mu a^b_\nu - \partial_\nu a^b_\mu + \epsilon^{bcd} a^c_\mu a^d_\nu \) is the SU(2) gauge field (ordered phase) and \( \phi^a \) is a scalar field which describes 2 and 4-points Green’s functions of SU(3)/SU(2) coset fields (disordered phase) via a relationship similar to eq. (8) i.e. \( G^{mn}(x,y) = \frac{1}{4} f^{mpb} f^{npc} \phi^b(x) \phi^c(y) \). A numerical investigation of the effective Lagrangian of eq. (9) shows that there are finite energy solutions whose profiles for the longitudinal color electric field and transversal electric and magnetic fields are given on Fig. 1B.

In conclusion by applying a Heisenberg-like quantization method to pure Yang-Mills theory we are able to construct effective Lagrangians that have only scalar fields or a mixture of scalar plus gauge fields. Both of these systems have finite energy solutions which are phenomenologically interesting. The completely disordered phase Lagrangian, containing only effective scalar fields, had finite energy solutions which could be interpreted as glueballs. The second case had a Lagrangian with both ordered (the SU(2) gauge field) and disordered (the effective scalar field) phases. This system had finite energy flux tube-like solutions.

REFERENCES


FIGURE 1. A. The energy density of glueball/soliton solution. B. The color electric \( E_x^3(x), E_y^1(x) \) and magnetic \( H_\phi^3(x) \) fields